Quiz 1: Make up

Tuesday, January 31, 2023 10:05 PM

1. (*25 points*) Determine the velocity of D at the instant shown using: (i) vector analysis (15 points) and (ii) scalar method using the instantaneous center (10 points).

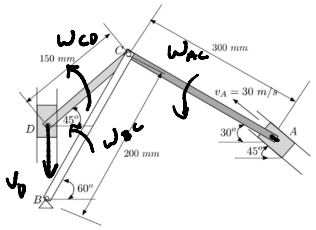


Figure 1: Problem 1

i) $V_D = V_C + \omega_{CD} \times r_{CD}$

$V_A = V_C + \omega_{AC} \times r_{CA}$

$V_C = \omega_{BC} \times r_{BC}$

$r_{CA} = 0.3\cos(30)\,\hat{\imath} - 0.3\sin(30)\,\hat{\jmath}$

$r_{CD} = -0.15\cos(45)\,\hat{\imath} - 0.15\sin(45)\,\hat{\imath}$

$r_{BC} = 0.2\cos(60)\,\hat{\imath} + 0.2\sin(60)\,\hat{\jmath}$

$V_A = -30\cos(45)\,\hat{\imath} + 30\sin(45)\,\hat{\jmath}$

$\omega_{BC} = \omega_{BC}\hat{k}$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & \omega_{BD} \\ 0.1 & 0.1\sqrt{3} & 0 \end{vmatrix} = -0.1\sqrt{3}\,\omega_{BC}\,\hat{\imath} + 0.1\,\omega_{BC}\,\hat{\jmath} = V_C$$

$$\omega_{AC} \times r_{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & \omega_{AC} \\ \frac{0.3\sqrt{3}}{2} & -0.15 & 0 \end{vmatrix} = 0.15\,\omega_{AC}\,\hat{\imath} + \frac{0.3\sqrt{3}}{2}\,\omega_{AC}\,\hat{\jmath}$$

$$-30\cos(45)\,\hat{\imath} + 30\sin(45)\,\hat{\jmath} = -0.1\sqrt{3}\,\omega_{BC}\,\hat{\imath} + 0.1\,\omega_{BC}\,\hat{\jmath} + 0.15\,\omega_{AC}\,\hat{\imath} + \frac{0.3\sqrt{3}}{2}\,\hat{\jmath}$$

$$-30\frac{\sqrt{2}}{2} = -0.1\sqrt{3}\,\omega_{BC} + 0.15\,\omega_{AC}$$

I don't care about ω_{AC}

$$30\frac{\sqrt{2}}{2} = 0.1\,\omega_{BC} + \frac{0.3\sqrt{3}}{2}\,\omega_{AC}$$

$$\left(30\frac{\sqrt{2}}{2} - 0.1\,\omega_{BC}\right)\frac{2}{0.3\sqrt{3}} = \omega_{AC}$$

$$-30\frac{\sqrt{2}}{2} = -0.1\sqrt{3}\,\omega_{BC} + 0.15\left(\frac{2}{0.3\sqrt{3}}\right)\left(30\frac{\sqrt{2}}{2} - 0.1\,\omega_{BC}\right)$$

$$-30\frac{\sqrt{2}}{2} = -0.1\sqrt{3}\,\omega_{BC} + \frac{1}{\sqrt{3}}\left(30\frac{\sqrt{2}}{2} - 0.1\,\omega_{BC}\right.$$

$$-15\sqrt{2} = -0.1\sqrt{3}\,\omega_{BC} + \frac{15\sqrt{2}}{\sqrt{3}} - \frac{0.1}{\sqrt{3}}\,\omega_{BC}$$

$$-15\sqrt{2} = -0.1\sqrt{3}\,\omega_{BC} + \frac{15\sqrt{2}}{\sqrt{3}} - \frac{0.1}{\sqrt{3}}\,\omega_{BC}$$

$$\omega_{BC}\left(\frac{0.1}{\sqrt{3}} + 0.1\sqrt{3}\right) = \frac{15\sqrt{2}}{\sqrt{3}} + 15\sqrt{2}$$

$$\omega_{BC} = \frac{\frac{15\sqrt{2}}{\sqrt{3}} + 15\sqrt{2}}{\left(\frac{0.1}{\sqrt{3}} + 0.1\sqrt{3}\right)} = 144.89 \text{ rad/s}$$

$$v_C = -25.0955\,\hat{i} + 14.4889\,\hat{j}$$

$$v_D = -25.0955\,\hat{i} + 14.4889\,\hat{j} + \omega_{DC} \times r_{DC}$$

$$\omega_{DC} \times r_{DC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega_{DC} \\ -0.15\frac{\sqrt{2}}{2} & -0.15\frac{\sqrt{2}}{2} & 0 \end{vmatrix} = 0.15\frac{\sqrt{2}}{2}\omega_{DC}\,\hat{i} - \frac{0.15\omega_{DC}}{2}\,\hat{j}$$

$$0\,\hat{i} + v_D\,\hat{j} = \left(-25.0955 + 0.15\frac{\sqrt{2}}{2}\omega_{DC}\right)\hat{i} + \left(14.4889 - \frac{0.15\omega_{DC}}{2}\right)\hat{j}$$

$$25.0955 = 0.15\frac{\sqrt{2}}{2}\,\omega_{DC}$$

$$\omega_{DC} = 236.6 \text{ rad/s}$$

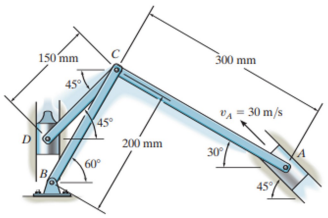
$$v_D = 14.4889 - \frac{0.15(236.6)}{2} = -10.6$$

$$v_D = -10.6 \text{ m/s}\,\hat{j}$$

ii) Scalar Method

16–87.

The mechanism shown is used in a riveting machine. It consists of a driving piston A, three members, and a riveter which is attached to the slider block D. Determine the velocity of D at the instant shown, when the piston at A is traveling at $v_A = 30$ m/s.



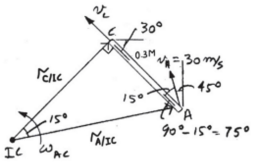
SOLUTION

Link AC:

$$\omega_{AC} = \frac{v_A}{r_{A/IC}} = \frac{30}{\left(\frac{0.3}{\sin 15°}\right)} = 25.88 \text{ rad/s}$$

$$v_C = \omega_{AC} r_{C/IC} = (25.88)\left(\frac{0.3}{\tan 15°}\right) = 28.98 \text{ m/s}$$

Link DC:



$$\frac{r_{C/IC}}{\sin 45°} = \frac{0.15}{\sin 120°}$$

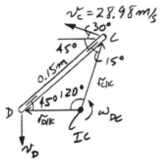
$$r_{C/IC} = 0.1225 \text{ m}$$

$$\frac{r_{D/IC}}{\sin 15°} = \frac{0.15}{\sin 120°}$$

$$r_{D/IC} = 0.04483 \text{ m}$$

$$\omega_{DC} = \frac{v_C}{r_{C/IC}} = \frac{28.98}{0.1225} = 236.60 \text{ rad/s}$$

$$v_D = \omega_{DC} r_{D/IC} = 236.60(0.04483)$$

$$v_D = 10.6 \text{ m/s} \downarrow$$

Ans.